Study on Techniques for Solving Constrained Optimization Problem

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ABSTRACT

The foundation of optimization techniques can be traced from 300 BC when Euclid recognized the minimum distance between two points to be length of straight line amalgamation the two. He also proved that a square has the greatest area among the rectangles with given total length of edges. Heron proved in 100 BC that light travels between two points through the path with shortest length when reflecting from a mirror. This paper studies on Techniques for Solving Constrained Optimization Problem.

KEYWORDS: Constrained, Optimization Techniques,

1. INTRODUCTION

Constrained optimization

We have seen how to characterize optimal solutions in constrained optimization

- **KKT optimality conditions** include the balance of forces $(-\nabla f(x^*), \nabla g_i(x^*), i \in I \text{ and } \nabla h_j(x^*))$ and complementarily conditions $(\mu_i g_i(x^*) = 0 \ \forall i)$
- **Regularity of x^* need to be assumed**

Methods for constrained optimization

Many methods utilize knowledge about the constraints – Linear inequalities or linear equalities – Nonlinear inequalities or equalities

For example, if a linear constraint is active at some point, you know that by taking steps along the direction of the constraint, it remains active

For nonlinear constraints, you don't have such a direction

Methods for constrained optimization can be characterized based on how they treat constraints.

Classification of the methods

- **Indirect methods:** the constrained problem is converted into a sequence of unconstrained problems whose solutions will approach to the solution of the constrained problem, the intermediate solutions need not to be feasible
- **Direct methods:** the constraints are taking into account explicitly, intermediate solutions are feasible

2. REVIEW OF LITERATURES

Prior to the development of analytics of varieties, the optimization issues like, deciding ideal measurements of wine barrel in 1615 by J. Kepler, a proof that light goes between two focuses in insignificant time in 1657 by P. De Fermat were settled. I. Newton (1660s) and G.W. von Leibniz (1670s) made scientific examination that structures the premise of analytics of variety. L. Euler's distribution in 1740 started the examination on general hypothesis of math of varieties. The technique for optimization for compelled issues, which include the expansion of obscure multipliers, wound up noticeably known by the name of its innovator, J. L. Lagrange. Cauchy made the main utilization of the slope technique to take care of unconstrained optimization issues in 1847. G. Dantzig exhibited Simplex technique in 1947. N. Karmarkar's polynomial time calculation in 1984 starts a blast of inside point optimization strategies. The progression in arrangement strategies came about a few very much characterized new territories in improvement techniques. The direct and non-straight limitations emerging in streamlining issue can be effectively taken care of by punishment technique. In this strategy at least few articulations are added to make target work less ideal as the arrangement approaches a requirement.

An Optimization Problem The fundamental segments of an advancement issue are: Objective Function A target work communicates at least one amounts which are to be limited or boosted. The improvement issues may have a solitary target capacity or more target capacities. Generally the diverse goals are not good. The factors that upgrade one target might be a long way from ideal for the others. The issue with multi-targets can be reformulated as single target issues by either framing a weighted blend of the diverse goals or by regarding a portion of the destinations as imperatives. Factors An arrangement of questions, which are fundamental, are called factors. The factors are utilized to characterize the target capacity and requirements. One can't pick plan variable discretionarily, they need to fulfil certain predefined utilitarian and different necessities. The plan factors can be constant, discrete or Boolean. Limitations An arrangement of requirements are those which enable the questions to go up against specific esteems yet prohibit others. They are conditions that must be fulfilled to render the outline to be doable. Once the plan factors, requirements, destinations and the connection between them have been picked, the optimization issue can be characterized [1-6].

Zhang et al. (2004) proposed the utilization of an occasional limitation taking care of mode in a PSO calculation. The principle thought is to influence intermittent duplicates of the hunt to space when the calculation begins the run. This intends to maintain a strategic distance from the scattering of particles that emerges from the utilization of the change administrator to particles lying on the limit between the doable and infeasible areas. This approach was approved receiving a low number of target work assessments (running from 28,000 to 140,000), and utilizing eight test issues. The outcomes delivered by the proposed approach were contrasted with deference with those created by conventional imperative dealing with procedures (i.e., punishment capacities), yet none is given regard to cutting edge developmental calculations intended for obliged look spaces.

In Toscano Pulido and Coello (2004) a basic imperative dealing with instrument in light of closeness of the particles in the swarm to the attainable area is consolidated into a PSO calculation. This approach additionally joins a transformation administrator (called turbulence), which changes the flight of the particles to various zones, meaning to look after decent variety. In the approval of this approach, the creators embraced a moderately huge populace estimate, and a low number of emphases, as to perform 340,000 target work assessments. The consequences of this approach were observed to be focused concerning those produced by cutting edge transformative calculations intended for compelled improvement (in particular, stochastic positioning **(Runarsson and Yao 2000),** homomorphous maps **(Koziel and Michalewicz 1999) and ASCHEA (Hamida and Schoenauer 2002))** when taking care of the thirteen test issues received in **(Runarsson and Yao 2000).**

Parsopoulos et al. (2005) proposed a Unified Particle Swarm Optimization approach, which was then adjusted to consolidate imperatives. This approach receives a punishment work, which utilizes data from the quantity of limitations damaged and the greatness of such infringement. Likewise, the plausibility of the best arrangements is saved. This approach was tried with four compelled designing optimization issues with promising outcomes. Be that as it may, no outcomes were given benchmark issues, which are ordinarily harder to settle.

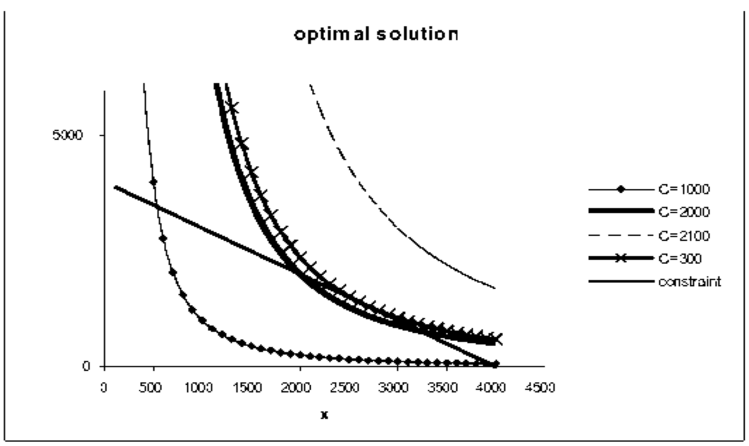
3. CONSTRAINED OPTIMIZATION: THE METHOD OF LAGRANGE MULTIPLIERS

Suppose the equation $p(x,y) = -2x^2 + 60x - 3y^2 + 72y + 100$ models profit when x represents the number of handmade chairs and y is the number of handmade rockers produced per week. The optimal (maximum) situation occurs when x = 15 and y = 12. However due to an insufficient labor force they can only make a total of 20 chairs and rockers per week (x + y = 20). So how many chairs and how many rockers will give the realistic maximum profit? We will come back to these questions shortly but first we will look at the following example. Using Level Curves and the constraint function to determine optimal points: A Company has a determined that its production function is the Cobb-Douglas function $f(x,y) = x^{\frac{2}{3}} y^{\frac{1}{3}}$ where x is the number of labor hours and y is the number of capitol units. The budget constraint for the company is given by 400000.=100y +100x a) If the company decides to spend $300, 000 on x then how much can be spent on y under this budget constraint?

Solution: If $300000 is spent on x then 100x = 300000 and x = 3000. There is $100000 left to spend on y therefore 100y = 100000 and y = 1000.

In this case total production will be f (3000, 1000) = $(3000)^{2/3}(1000)^{1/3}$ = (208)(10) = 2080 units. Suppose the amount spent on x is changed to $350,000. How will that change production? Following the same procedure as above x = 3500 and y = 500. The production f(3500, 500) then be 1829 units. Is there a way to determine values for x and y that will give the optimal production possible given this budget constraint? We will first try to find the optimal situation using level curves with the constraint function. It turns out that the global maximum or global minimum occurs where the graph of the constraint equation is tangent to one of the level curves of the original function.

In order to plot the level curves we must solve for y to get $\frac{f(x,y)}{x^{\frac{2}{3}}} = y^{\frac{1}{3}} \rightarrow \frac{C^3}{x^2} = y$ where C now represents values of f(x,y). Now we will plot y for various values of C. (The choice of values for C is determined mostly by common sense.) In this case we will let C = 1000, C = 2000 and C = 3000. After looking at the resulting level curves and constraint, it decide to add another value C = 2100. Now I can make a good guess After considering the graph below, we can guess the optimal value will occur for x = 2600 and y = 1400.



There is also an algebraic approach to finding the optimal solution given a certain constraint. We will use the method of Lagrange Multipliers to find the maximum situation in the problem above. In the proceeding sections you have learned how to use partial derivatives to find the optimal situation for a multivariable equation.

Example-

Amanda is getting a new dog. She wants to build a pen for her dog in the back yard. The pen will be rectangular using 200 feet of fence. Amanda plans to build the pen up against the wall of her house so that she will only need three sides of fence. She wants to build a pen with the maximum amount of area. Therefore we need to maximize the area equation $A = xy$. Since Amanda has a constraint in the amount of fencing she can use, we will use the LaGrange method and create the following equation. $L(x,y) = xy- \lambda(x+y-200)$.

Now find $Lx = 0$, $Ly = 0$ and solve for x and y to get that $x =\lambda$ and $y = 2\lambda$. Find $L\lambda =0$ and substitute in the values for x and y to put the equation in terms ofλ.

CONCLUSION

Now solve to$\lambda$and substitute in the values for x and y to put the equation. Therefore the optimal values are $x = 50$ and $y = 100$ feet of fencing.λGet that (Notice that if the constant 200 feet of fencing is increased by on foot to 201 feet then by marginal analysis the maximum value for the area will increase by 50 square feet.

REFERENCES

1. Bochenek, B. and Forys, P., 2006. Structural optimization for post-buckling behavior using particle swarms. Structural and Multidisciplinary Optimization, 32 (6), 521– 531.

2. Cagnina, L., Esquivel, S., and Gallard, R., 2004. Particle Swarm Optimization for sequencing problems: a case study. In: Proceedings of the 2004 IEEE Congress on Evolutionary Computation (CEC 2004). Portland, Oregon, USA. 20-23 June. IEEE Press, 536–541.
3. Eberhart, R.C. and Shi, Y., 2000. Comparing Inertia Weights and Constriction Factors in Particle Swarm Optimization. In: Proceedings of the 2000 IEEE Congress on Evolutionary Computation (CEC'2000). Piscataway, New Jersey, USA. 16-19 July IEEE Press, 84–88.
4. Hamida, S.B. and Schoenauer, M., 2002. ASCHEA: New Results Using Adaptive Segregational Constraint Handling. In: Proceedings of the Congress on Evolutionary Computation 2002 (CEC'2002). Honolulu, Hawaii. 12-17 May. IEEE Service Center, 884–889.
5. Knowles, J.D. and Corne, D.W., 2000. Approximating the Nondominated Front Using the Pareto Archived Evolution Strategy. Evolutionary Computation, 8 (2), 149–172.
6. Li, L.J., et al., 2007. A heuristic particle swarm optimizer for optimization of pin connected structures. Computers and Structures, 85 (7–8), 340–349.
7. Zhang, W.J. and Xie, X.F., 2003. DEPSO: Hybrid Particle Swarm with Differential Evolution Operator. In: Proceedings of the IEEE International Conference on Systems, Man and Cybernetics (SMC 2003), Vol. 4. Washington DC, USA. 5-8 October. IEEE, 3816–3821
8. Toscano-Pulido, G. and Coello Coello, C.A., 2004. A Constraint-Handling Mechanism for Particle Swarm Optimization. In: Proceedings of the Congress on Evolutionary

Computation 2004 (CEC 2004). Portland, Oregon, USA. 20-23 June. Piscataway, New Jersey: IEEE Service Center, 1396–1403.

9. Runarsson, T.P. and Yao, X., 2000. Stochastic Ranking for Constrained Evolutionary Optimization. IEEE Transactions on Evolutionary Computation, 4 (3), 284–294.
10. Koziel, S. and Michalewicz, Z., 1999. Evolutionary Algorithms, Homomorphous Mappings, and Constrained Parameter Optimization. Evolutionary Computation, 7 (1), 19–44.
11. Hamida, S.B. and Schoenauer, M., 2002. ASCHEA: New Results Using Adaptive Segregational Constraint Handling. In: Proceedings of the Congress on Evolutionary Computation 2002 (CEC'2002). Honolulu, Hawaii. 12-17 May. IEEE Service Center, 884–889.
12. Parsopoulos, K. and Vrahatis, M., 2005. Unified Particle Swarm Optimization for solving constrained engineering optimization problems. Advances in Natural Computation, Pt. 3. Lecture Notes in Computer Science Vol. 3612, 582–591.