Fibonacci Numbers in Action

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Abstract: An example of the power of math can be found in Fibonacci numbers. The Fibonacci numbers are sequences of numbers of the form: **0,1,1,2,3,5,8,13,...** These numbers are famous for possessing wonderful and amazing applications. The sequence plays a central role in elementary number theory. In mathematical terms, it is defined by the following recurrence relation:

$$F_n = F_{n-1} + F_{n-2} \text{ with } F_1 = F_2 = 1 \text{ and } F_0 = 0$$

The first number of the sequence is 0, the second number is 1, and each subsequent number is equal to the sum of the previous two numbers of the sequence itself. That is, after two starting values, each number is the sum of the two preceding numbers. The Fibonacci numbers appear in an amazingly variety of creations, both natural and people made. The numbers have very interesting properties, and keep popping up in many places in nature and art In this paper. The Fibonacci sequence also makes its appearance in many different ways within mathematics. In this paper we investigate some important applications of the Fibonacci numbers.

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Key Words: Fibonacci numbers, Fibonacci sequences, and Pascal's triangle.

1. Introduction. The Fibonacci numbers are a sequence of numbers named after Leonardo of Pisa, known as Fibonacci. Fibonacci's 1202 book *Liber Abaci* introduced the sequence to Western European mathematics, although the sequence had been previously described by Indian mathematics . Here are the First 20 Fibonacci numbers.

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}	F_{20}
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181	6765

Main Applications:

1. Applications of Fibonacci Numbers in Nature

Application 1: The Rabbit Breeding Problem (cause for the discover of Fibonacci numbers).

Start with a pair of rabbits, (one male and one female) born on January 1, 2007. Assume that all months are of equal length and that :

1. Rabbits begin to produce young two months after their own birth;

2. After reaching the age of two months, each pair produces a mixed pair, (one male, one female), and then another mixed pair each month thereafter; and

3. No rabbit dies.

How many pairs of rabbits will there be after one and half year ?

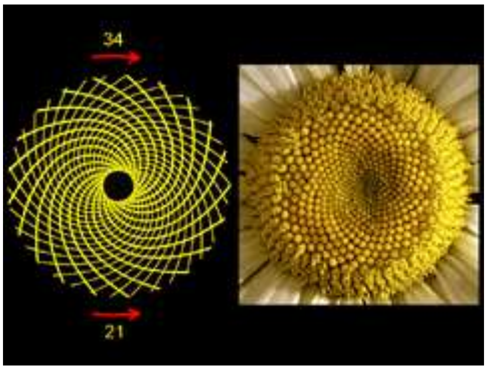
Note that at the end of the one and half year, there will be F_{18}=2584 pairs of rabbits, all resulting from the one original pair born on January 1 of 2007.

Note that the above rabbit breeding problem that caused Fibonacci to write about the sequence in *Liber abaci* may be unrealistic but the Fibonacci numbers really do appear in nature. For example, some plants branch in such a way that they always have a Fibonacci number of growing points. Flowers often have a Fibonacci number of petals, daisies can have 34, 55 or even as many as 89 petals[2].

Application 2: Flower Petals

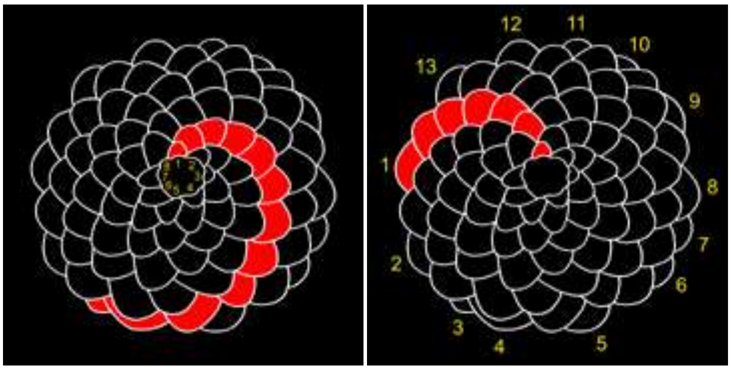
In nature, many flowering plants produce flowers with numbers of petals that correspond to Fibonacci numbers. For instance, the trillium has three petals, the wild rose has five petals, the delphinium has eight petals, and many other examples.

Application 3: Sunflower Seeds



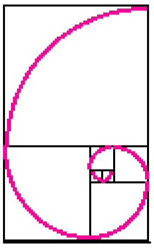
Sunflowers produce their seeds in tightly packed spirals. The number of clockwise and counterclockwise spirals are nearly always consecutive Fibonacci numbers. Notice above, there are 34 clockwise spirals and 21 counterclockwise spirals, which are consecutive Fibonacci numbers.

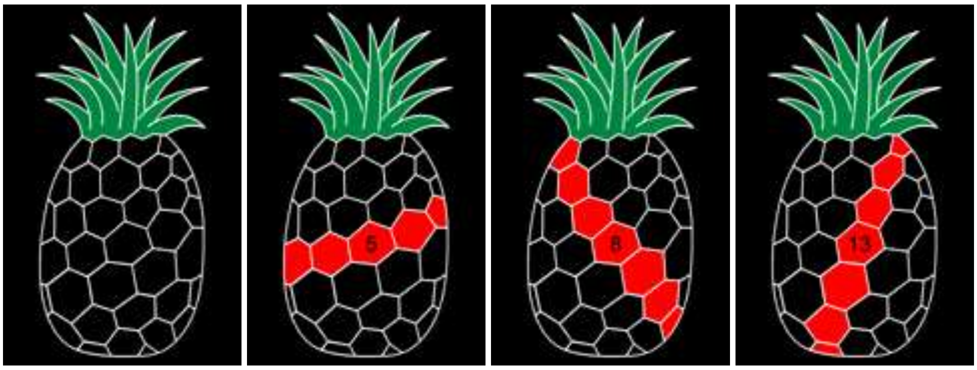
Application 4: Pinecones



Pinecones produce scales packed closely together in spirals. These spiral patterns often come in two sets, known as parastichies, one set clockwise, the other counter-clockwise. Many are found to be consecutive Fibonacci numbers. In the above picture, there are 13 clockwise spirals and 8 counter-clockwise spirals.

Application 5: Pineapples





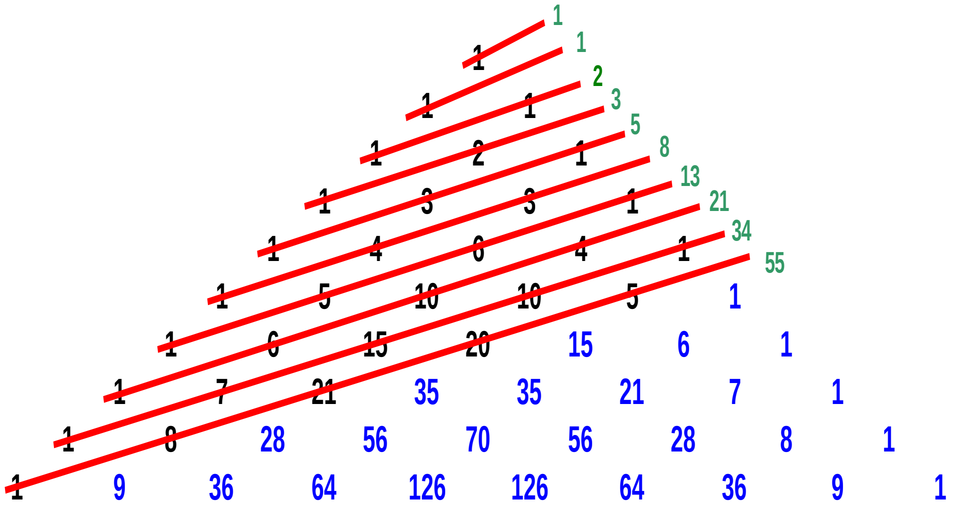
Pineapples exhibit multiple spiral patterns. Like pinecones, pineapples often produce parastichies; however, many pineapples exhibit three spiral patterns due to their roughly hexagonal scale pattern. Each of these spirals match consecutive Fibonacci numbers. Notice above with the nearly horizontal spirals, and the two steeper spirals – one clockwise and the other counter-clockwise. Three spirals are possible due to the hexagonal shape of a pineapple's scale.

Application 6: Seashells

When squares are arranged by size from two size 1, to size 3, size 5, size 8, size 13, size 21, and size 34, it forms the rectangle depicted to the left. A spiral can be formed with arcs connecting two opposite corners of each square. It should be noted that it is not actually a mathematical spiral. On the following page is a cross-sectional radiograph of a Nautilus shell, a member of the cephalopod family Nautilidae. The shells are formed in nearly perfect Fibonacci spirals.

2. Applications of Fibonacci Numbers in Mathematics

1. Fibonacci Numbers and Pascal's Triangle



2. The Golden Ratio.

In mathematics and the arts, two quantities are in the golden ratio if the ratio between the sum of those quantities and the larger one is the same as the ratio between the larger one and the smaller. The golden ratio is an irrational mathematical constant. It is denoted by ϕ. This value is obtained by equating the ratio between the sum of the successive terms and the larger one to the ratio of successive terms in the Fibonacci sequence as shown $\frac{1}{1},\frac{2}{1},\frac{3}{2},\frac{5}{3},\frac{8}{5},\ldots$ That is if m and n are two successive terms in the Fibonacci sequence, we have $\frac{n}{m}\cong\frac{m+n}{n}$

Theorem 1.. The Golden Ratios of the Fibonacci Sequence seem to be tending to a limit equal to $\frac{1+\sqrt{5}}{2}$.

Proof: Let m, n , and m + n be successive terms of the sequence.

Then we have $\frac{n}{m}\cong\frac{m+n}{n}$

$$\Rightarrow \frac{n}{m}\cong 1+\frac{m}{n}$$

Defining ϕ to be the limit of $\frac{n}{m}$, we have $\phi = 1+\frac{1}{\phi}$.

$\Rightarrow \phi^2 = \phi + 1 \Rightarrow \phi^2 - \phi - 1 = 0$. Using Quadratic Formula, we get $\phi = \frac{1+\sqrt{5}}{2}$.

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