Common Neighbourhood Domination in Some Operations on Fuzzy Graphs

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Abstract

In this paper, the concept of common neighbourhood $CN-dominating$ in some operations on fuzzy graphs and denoted by γ_{cn} introduced and investigated the bound of of some operations on fuzzy graphs are obtained such as union, join, Cartesian product and composition.

Keywords:- fuzzy graph, common neighbourhood.
AMS Mathematics Classification (2010), 03E72, 68R10, 68R05.

1 Introduction

A. Alwardi, N.D Soner and Karam Ebadi [2] introduced and studied common neighbourhood dominating set in graph $CN-dominating$. The fuzzy relations between fuzzy sets were also considered by Rosenfeld [10] and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. The concept of domination number in fuzzy graphs was investigated by A. Somasundaram and S. Somasundaram [11]. Abdullah Q. AL-Mekhlafi, Mahiuob M.Q Shubatah and Saqr H..ALemrany [1] introduced and studied common neighbourhood dominating set in fuzzy graph $CN-domination$. A. Nagorgani and S.R. Latha [4], introduced some operations on fuzzy graph. In this paper, we introduce and investigate the concept of common neighbourhood domination number in some operations on fuzzy graphs, we obtain the bounds of the common neighborhood domination number in some operations on fuzzy graphs like union, join, Cartesian product and composition.

2 Preliminaries

In this section, we review some basic definitions related to fuzzy graphs and common neighborhood domination in fuzzy graph.

Definition 2.1. *[1]* Let $G = (\mu, \rho)$ be fuzzy graph with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$. For $i \neq j$, the common neighborhood of the vertices v_i and v_j, denoted by $\Gamma(v_i, v_j)$, is the set of vertices, different from v_i and v_j, which are adjacent to both v_i and v_j.

Definition 2.2. *[10]* The edge between any vertices u and V in G is called effective edge if $(\rho(u, v) = \mu(u) \wedge \mu(v))$. The vertex v is adjacent to a vertex u, if they reach between the effective edge.

Definition 2.3. *[10]* Two vertices v_i and v_j are said to be neighbors in a fuzzy graph G, Then $N(v) = \{u \in V : \rho(u, v) = \mu(u) \wedge \mu(v)\}$ is called the open neighborhood set of v and $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood set of v.

Definition 2.4. *[1]* Let $G = (\mu, \rho)$ be a fuzzy graph and let u and v are any two adjacent vertices in G such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ and $|\Gamma(u, v)| > 0$, then we say u is common neighbourhood adjacent (CN-adjacent) to v or u is CN-dominate v.

Definition 2.5. *[11]* Let $G = (\mu, \rho)$ be a fuzzy graph on V. Let $u, v \in V$. We say that u dominates v in G if $\rho(u, v) = \mu(u) \wedge \mu(v)$. A subset D of V is called a dominating set in G if for every $v \in V - D$, there exists $u \in D$ such that u dominates v.

Definition 2.6. *[11]* The minimum fuzzy cardinality of dominating sets in G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set D of a fuzzy graph G is said to be a minimal dominating set if no proper subset of S is dominating set of G.

Definition 2.7. *[1]* Let $G = (\mu, \rho)$ be a fuzzy graph a subset D of V is called common neighbourhood dominating set $(CN - dominating)$ if for every vertex $v \in V - D$ there exists a vertex $u \in D$, such that $\rho(u, v) = \mu(u) \wedge \mu(v)$ and $|\Gamma(u, v)| > 0$, where $\Gamma(u, v)$ is the number of common neighbourhood between the vertices u and v, the common neighbourhood domination number $CN - dominating$ number is the minimum fuzzy cardinality taken over all minimal common neighbourhood dominating sets of G and is donated by $\gamma_{cn}(G)$ or γ_{cn}.

Definition 2.8. *[1]* Let $G = (\mu, \rho)$ be a fuzzy graph a common neighbourhood dominating set D is said to be minimal common neighbourhood dominating set if $D - \{u\}$ is not common neighbourhood dominating set of G for all $v \in D$. A minimal common neighbourhood dominating set D is called minimum common neighbourhood dominating set of G if $|D| = \gamma_{cn}(G)$ and is denoted by $\gamma_{cn} - set$.

3 Mine Result

In this section, we introduce and study the concept of common neighbourhood domination in some operations on fuzzy graphs such as the union, the join, the cartesian product and the composition.

Definition 3.9. *[13]* Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two fuzzy graphs on V_1, V_2 respectively with $V_1 \cap V_2 = \phi$. The union of G_1 and G_2 and by $G_1 \cup G_2$ is the fuzzy graph on $V_1 \cup V_2$ and defined $G = G_1 \cup G_2 = \{(\mu_1 \cup \mu_2), (\rho_1 \cup \rho_2\}$. Where

$(\mu_1 \cup \mu_2)(x) = \begin{cases} \mu_1(x) & if x \in V_1 \\ \mu_2(x) & if x \in V_2 \end{cases}$. $(\rho_1 \cup \rho_2)(xy) = \begin{cases} \rho_1(xy) & if xy \in E_1 \\ \rho_2(xy) & if xy \in E_2 \\ 0 & otherwise \end{cases}$.

Theorem 3.10. *Let G_1 and G_2 be two vertices disjoint fuzzy graphs, Then*

$$\gamma_{cn}(G_1 \cup G_2) = \gamma_{cn}(G_1) + \gamma_{cn}(G_2).$$

Proof. Let G_1 and G_2 be any two fuzzy graphs with $\gamma_{cn}(G_1)$ and $\gamma_{cn}(G_2)$, respectively. since $\gamma(G_1) \leq \gamma_{cn}(G_1)$ and $\gamma(G_2)$. Then
$\gamma(G_1) + \gamma(G_2) \leq \gamma_{cn}(G_1) + \gamma_{cn}(G_2)$ Then

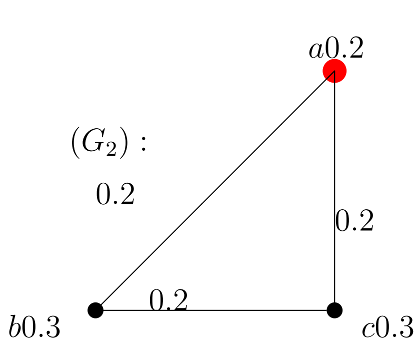
$$\gamma(G_1 \cup G_2) \leq \gamma_{cn}(G_1 \cup G_2) = \gamma_{cn}(G_1) \cup \gamma_{cn}(G_2).$$

Therefore

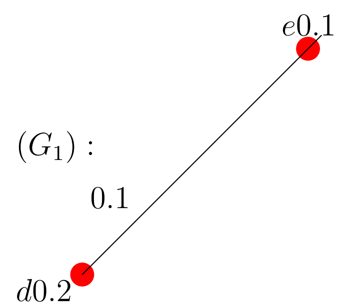
$$\gamma_{cn}(G_1 \cup G_2) = \gamma_{cn}(G_1) + \gamma_{cn}(G_2).$$

□

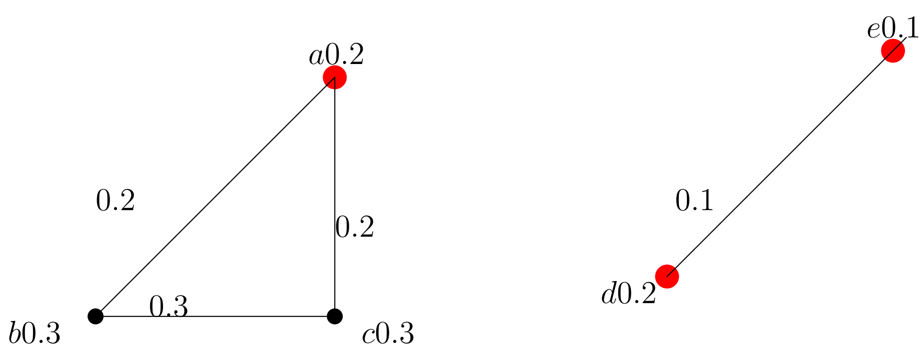
Example 3.11. Consider the fuzzy graphs G_2 and G_1 given in the Figures $3.1a$ and $3.1b$ respectively.



$Fig\ 3.1a$



$Fig\ 3.1b.$



$Fig\ 3.1c : (G_1 \cup G_2)$

The union of G_1 and G_2 is shown in Figure $(3.1c)$, we see that $S_1 = \{a\}$ is a $\gamma_{cn} - set$ of G_1, $S_2 = \{d, e\}$ is a $\gamma_{cn} - set$ of G_2 and $S = S_1 \cup S_2$ is a $\gamma_{cn} - set$ of $\gamma_{cn}(G_1 \cup G_2) = |S_1 + S_2| = |\{a\} + \{d, e\}| = 0.2 + 0.3 = 0.5$.

Definition 3.12. *[13]* Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two fuzzy graphs on V_1, V_2 respectively with $V_1 \cap V_2 = \phi$. The union of G_1 and G_2 and by $G_1 \cup G_2$ is the fuzzy graph on $V_1 \cup V_2$ and defined $G = G_1 + G_2 = \{(\mu_1 + \mu_2), (\rho_1 + \rho_2)\}$.
Where

$$(\mu_1 + \mu_2)(x) = \begin{cases} \mu_1(x) & if \quad x \in V_1 \\ \mu_2(x) & if \quad x \in V_2 \end{cases}.$$

$$(\rho_1+\rho_2)(xy)=\begin{cases}\rho_1(xy) & if \quad xy\in E_1\\ \rho_2(xy) & if \quad xy\in E_2\\ min\{\rho_1(x),\rho_2(y)\} & if \quad xy\in E^{`}\end{cases}.$$

where $E^{`}$ is the set of all edge joining the vertices of V_1 and V_2.

Theorem 3.13. For any two fuzzy graphs G_1 and G_2 such that $G_1 \cap G_2 = \phi$. Then

$$\gamma_{cn}(G_1+G_2)\leq min\{\gamma(G_1),\gamma(G_2)\}.$$

Proof. *Let D_1, D_{cn1} D_2 and D_{cn2} be a dominating sets and $CN-dominating$ sets of G_1 and G_2, respectively. Then $|D_1|\leq|D_{cn1}|$ and $|D_2|\leq|D_{cn2}|$. Since $D_{cn1}\subseteq V_1$ and $D_{cn2}\subseteq V_2$. Then $D_{cn1}\subseteq V_1\cup V_2$ and $D_{cn2}\subseteq V_1\cup V_2$. Therefore, D_{cn1} and D_{cn2} are $CN-dominating$ sets in G_1+G_2. Since D_1 is dominating set of G_1+G_2. Then by Definition of join a vertex u in D_1 has common neighborhood with every vertices of a fuzzy graph G_1+G_2. Therefore, D_1 is $CN-dominating$ set of G_1+G_2. Similarly, D_2 is $CN-dominating$ set of fuzzy graph G_1+G_2. Since D_{cn1} and D_{cn2} are $CN-dominating$ sets of G_1+G_2 such that $|D_1|\leq|D_{cn1}|$ and $|D_2|\leq|D_{cn2}|$. Hence,*

$$\gamma_{cn}(G_1+G_2)\leq min\{\gamma(G_1),\gamma(G_2)\}.$$

□

Example 3.14. Consider the fuzzy graphs G_1 and G_2 given in the Figures $3.2a$ and $3.2b$, respectively. Such that all edges in G_1 and G_2 are effective.

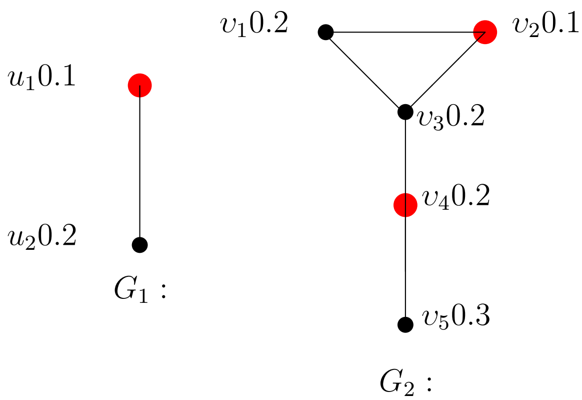


Fig 3.2*a*.

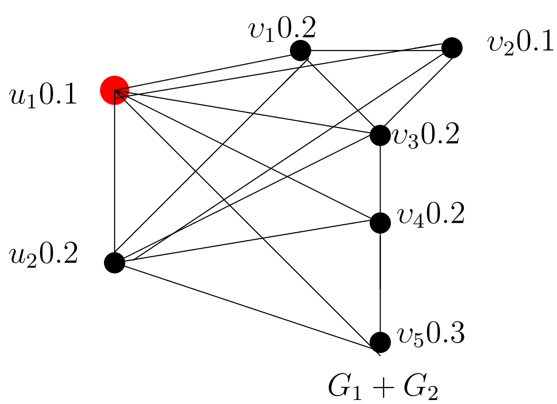


Fig 3.2*b*

The join of G_1 and G_2 given in Figure (3.2b), we see that $S_1 = \{u_1\}$ and $S_2 = \{v_2, v_4\}$ are dominating sets of G_1 and G_2, respectively. Then $\gamma_{cn}(G_1 + G_2) = min\{\gamma(G_1), \gamma(G_2)\} = 0.1$

Definition 3.15. *[4]* Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two fuzzy graphs on V_1, V_2 respectively with $V_1 \cap V_2 = \phi$. The cartesian product of G_1 and G_2 and by $G_1 \times G_2$ is the fuzzy graph on $V_1 \times V_2$ and defined $G = G_1 \times G_2 = \{(\mu_1 \times \mu_2), (\rho_1 \times \rho_2)\}$. such that

$(\mu_1 \times \mu_2)(xy) = min\{\mu_1(x), \mu_2(y)\} \qquad \forall x, y \in V_1 \times V_2.$

$(\rho_1 \times \rho_2)((xy_1)(xy_2)) = min\{\mu_1(x), \rho_2(y_1y_2)\} \qquad \forall x \in V_1, y_1y_2 \in E_2.$

$(\rho_1 \times \rho_2)((x_1y)(x_2y)) = min\{\rho_1(x_1x_2), \mu_2(y)\} \qquad \forall y \in V_2, x_1x_2 \in E_1.$

Theorem 3.16. Let G_1 and G_2 be two fuzzy graphs on V_1 and V_2 respectively, with $G_1 \cap G_2 = \phi$. Then

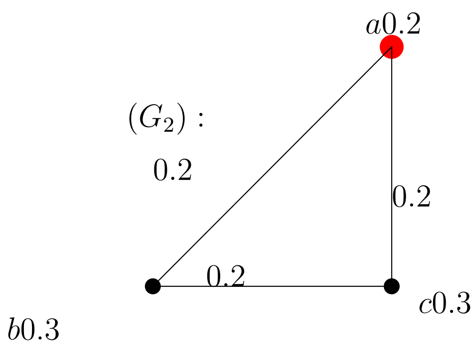
$$\gamma_{cn}(G_1 \times G_2) \leq min\{|D_{cn1} \times V_2|, |V_1 \times D_{cn2}|\}.$$

Proof. *Let D_1 and D_2 be two a dominating sets of G_1 and G_2 respectively and D_{cn1} and D_{cn2} be two a $CN-$dominating sets of G_1 and G_2 respectively. We went prove that $D_{cn1} \times V_2$ is $CN-$dominating set of $G_1 \times G_2$. Let $(x, y) \notin D_{cn1} \times V_2$. Hence $x \notin D_{cn1}$ since D_{cn} is $CN-$dominating set of G_1 there exist vertex $v \in D_{cn1}$ such that $\forall y \in V - D_{cn1}$. Than $|Gamma(v, y)| > 0$ and*
$\rho(v, y) = \mu(v) \wedge \mu(y)$
$\rho(x, y), (v, y) = min\{\rho(x, v) \wedge \mu(y)\}$
$= min\{\mu(x) \wedge \mu(v) \wedge \mu(y)\}$
$= min\{(\mu(x) \wedge \mu(y))min(\mu(v) \wedge \mu(y))\}$
$= \mu(xy) \wedge (vy)$. *Then $(x, y) \in D_{cn1} \times V_2$. Therefore, $D_{cn1} \times V_2$ is $CN-$dominating of $G_1 \times G_2$. Similarly $V_1 \times D_{cn2}$ is $CN-$dominating of $G_1 \times G_2$. Now we prove that $D_{cn1} \times V_2$ and $V_1 \times D_{cn2}$ are minimal. Suppose that $D_{cn1} \times V_2$ is not minimal there exist a vertex $(v, u) \in D_{cn1} \times V_2$ such that $\{D_{cn1} \times V_2 - (v, u)\}$ is $CN-$dominating set of $G_1 \times G_2$. This implies $D_{cn1} - \{v\}$ is $CN-$dominating set of G_1 that is contradict to our assumption D_{cn1} is minimal. Therefore, $D_{cn1} \times V_2$ is minimal dominating set of $G_1 \times G_2$. Similarly, $V_1 \times D_{cn2}$ is minimal dominating set of $G_1 \times G_2$. Hence*

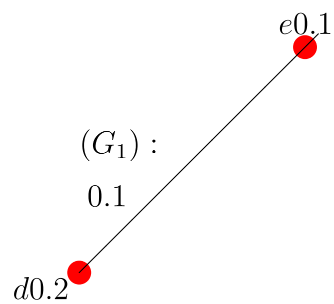
$$\gamma_{cn}(G_1 \times G_2) \leq min\{|D_{cn1} \times V_2|, |V_1 \times D_{cn2}|\}.$$

□

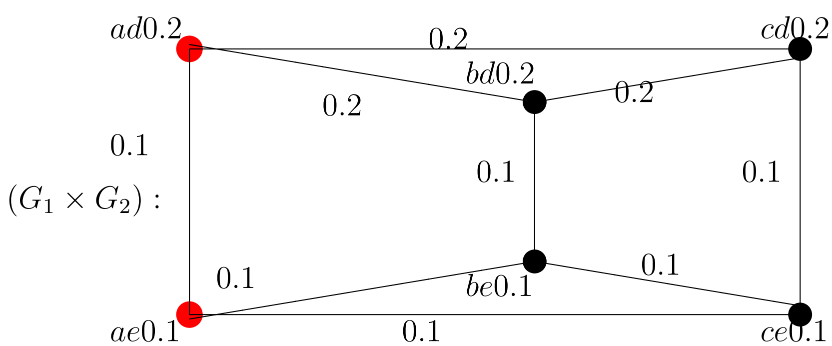
Example 3.17. Consider the fuzzy graphs G_2 and G_1 given in the Figures $3.3a$ and $3.3b$.



$Fig\ 3.3a$



$Fig\ 3.3b.$



$Fig\ 3.3c.$

The Cartesian product of G_1 and G_2 given in Figure $(3.3c)$, we see that $S_1 = \{e, d\}$ is a $\gamma_{cn} - set$ of G_1 and $S_2 = \{a\}$ is a $\gamma_{cn} - set$ of G_2. Then

$$\gamma_{cn}(G_1 \times G_2) = min\{|S_1 \times V_2|, |V_1 \times S_2|\} = min\{|\{ad, ae\}|, |\{ad, ae\}|\} = \{|ad, ae|\} = 0.3$$

Definition 3.18. [4] Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two on V_1, V_2 respectively with $G_1 \cap G_2 = \phi$. The composition of G_1 and G_2 and by $G_1 \circ G_2$ is the fuzzy graph on $V_1 \circ V_2$ and defined $G = G_1 \circ G_2 = (G_1[G_2]) = (V_1 \times V_2, V_1 E_2 + V_2^2 E_1)$, such that

$(\mu_1 \circ \mu_2)(xy) = min\{\mu_1(x), \mu_2(y)\} \qquad \forall x, y \in V_1 \times V_2$

$(\rho_1 \circ \rho_2)((xy_1)(xy_2)) = min\{\mu_1(x), \rho_2(y_1y_2)\} \qquad \forall x \in V_1, y_1y_2 \in E_2.$
$(\rho_1 \circ \rho_2)((x_1y)(x_2y)) = min\{\rho_1(x_1x_2), \mu_2(y)\} \qquad \forall y \in V_2, x_1x_2 \in E_1.$
$(\rho_1 \circ \rho_2)((x_1y_1)(x_2y_2)) = min\{\rho_1(x_1x_2), \mu_2(y_1 \wedge \mu_2(y_2))\} \forall y_1y_2 \in V_2, x_1x_2 \in E_1.$
Such that $y_1 \neq y_2$.

Theorem 3.19. For any two fuzzy graphs G_1 and G_2, such that $G_1 \cap G_2 = \phi$. Then

$$\gamma_{cn}(G_1 \circ G_2) \leq |D_1 \times D_2|.$$

Proof. *Let* D_1, D_{cn1}, D_2 *and* D_{cn2} *be a dominating sets and* $CN-$*dominating sets of* G_1 *and* G_2, *respectively. Then* $|D_1| \leq |D_{cn1}|$ *and* $|D_2| \leq |D_{cn2}|$. *Therefore,* $|D_1 \times D_2| \leq |D_1 \times D_{cn1}|$. *Let* $G = G_1 \circ G_2$ *be a fuzzy graph. Then* $\gamma(G) \leq \gamma_{cn}(G)$ *Since* D_1 *is dominating set of* G. *Then by Definition of composition a vertex* u *in* D_1 *has common neighborhood with a vertex* v *of a fuzzy graph* G *Therefore,* D_1 *is* $CN-$*dominating set of* G. *Similarly,* D_2 *is* $CN-$*dominating set of fuzzy graph* G. *Since* D_{cn1} *and* D_{cn2} *are* $CN-$*dominating sets of* G *such that* $|D_1| \leq |D_{cn1}|$ *and* $|D_2| \leq |D_{cn2}|$. *Since* $\gamma(G) \leq \gamma_{cn}(G)$. *Then by Theorem (3.6) in [5]. Hence*

$$\gamma_{cn}(G_1 \circ G_2) \leq |D_1 \times D_2|.$$

□

Example 3.20. The G_2 and G_1 are two fuzzy graphs given in the Figures $3.4a$ and $3.4b$.

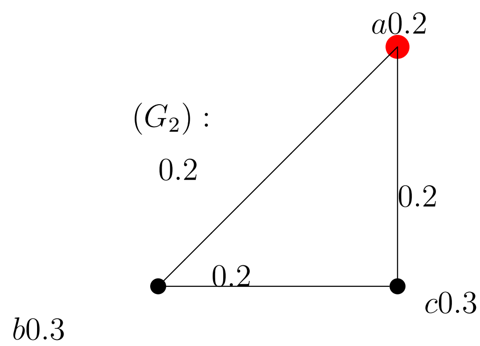


Fig 3.4*a*

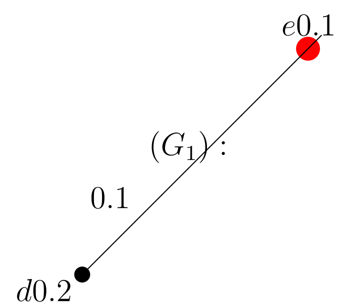
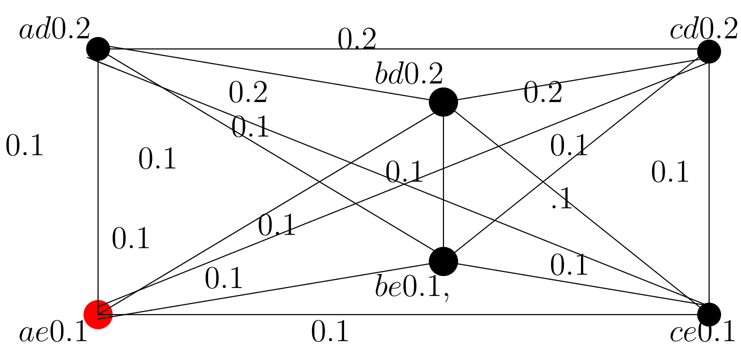


Fig 3.4*b*.



$Fig\ 3.4c : G_1 \circ G_2$

The composition of G_1 and G_2 given in Figure (3.4c), we see that $D_1 = \{a\}$ is a $\gamma - set$ of G_1, $D_2 = \{e\}$ is a $\gamma - set$ of G_2. Therefore, the $CN - dominating$ of $G_1 \circ G_2$ is $|D_1 \times D_2| = |ae| = 0.1$.

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