CONNECTED DOMINATING SET IN INTERVAL-VALUED FUZZY GRAPHS

Faisal M. AL-Ahmadi[1*] and Mahiuob M. Q. Shubatah[2]
*[*1]Departement of Mathematics, Sheba Region University, Faculty of Education, Arts and Science Marib, Yemen*
[2]Department of Mathematics, Faculty of Science and Education, Albaydha University, Albaydha, Yemen
*[*1]E-mail: faissalalahmadi@gmail.com. [2]E-mail: mahioub70@yahoo.com*

***Corresponding Author: -**
faissalalahmadi@gmail.com

Abstract: -
The connected domination is one of the important issues in mathematics and other science. In This paper, connected domination in interval-valued fuzzy graphs is defined and studied. Some bounds on connected domination number $\gamma_c(G)$ is provided for sev- eral interval-valued fuzzy graphs. Furthermore, the relationship of $\gamma_c(G)$ with some other known parameters in interval-valued fuzzy graphs are investigated with some suit- able examples.

Keywords: - *Interval-valued fuzzy graph, connected dominating set, connected domi-nation number.*

2010 Mathematics Subject Classification: (03E72), (05C69), (05C72).

1

1 INTRODUCTION

Rosenfeld [7] in (1975) introduced The nation of fuzzy graphs and widely definitions for each fuzzy vertex, fuzzy edges and several fuzzy analogs of graph theoretic concepts like paths, cycles, connectedness etc. Zadeh in (1975)[13] introduced another detailed the idea of interval-valued fuzzy sets as an extension of fuzzy sets, which gives more precise tool to model uncertainty in real-life situations. Interval-valued fuzzy sets have been widely used in many areas of science and engineering. In (2011) Akram and Dudek [1] introduced the notion of an interval-valued fuzzy graph and studied several properties and operations on interval-valued fuzzy graphs. In (1979) the Connected Domination in graphs was dis- cussed by Sampathkumar and Walikar [8]. The concept of domination in fuzzy graphs was investigated by A. SomaSundaram and S. SomaSundaram [11] further A. SomaSundaram presented the concepts of independent domination, total domination, connected domination of fuzzy graphs [12]. The concept of connected domination in fuzzy graphs using strong arcs was introduced by Manjusha and Sunitha in (2015) [5]. Sarala and Janaki in (2019) [9] discussed the concept of connected domination number in fuzzy digraphs. The concepts of domination in interval-valued fuzzy graphs was investigated by Pradip Debnath in (2013) [3], Because of the Large area for dominating applications in the real-life, like computer networks and the Internets network. In this paper we introduce and study the concepts of connected domination number in interval-valued fuzzy graphs. We obtain many results related to these concepts. Finally the relationship between this concepts and the others in interval-valued fuzzy graph will be obtained with some examples.

2 Preliminaries

In this section we will review some basic concepts that have a relationship to our subject.

Definition 2.1. *[7]* Let $G = (V, \mu, \rho)$ is called a fuzzy graph of crisp graph $G^* = (\mu^*, \rho^*)$, where V is a vertex set of G, μ is a fuzzy subset of V and ρ is a fuzzy relation on μ or ρ the fuzzy edge set of G and $\rho(u, v) \leq \mu(x) \wedge \mu(y)$ for u and v in V.

Definition 2.2. *[13]* A subset A of a vertex set V is called interval valued fuzzy set and it is denoted by $A = \{u, [\mu_1/(u), \mu_2(u)] : u \in V\}$, where the function $\mu_1 : V \to [0, 1]$ and $\mu_2 : V \to [0, 1]$, such that $\mu_1(u) \leq \mu_2(u)$ for $u \in V$. If $G^* = (V, E)$ is a crisp graph, then by an interval valued fuzzy relation $\rho = (\rho_1, \rho_2)$ on V, we mean an interval valued fuzzy on E, such that and

$$\rho_1(u, v) \leq \mu_1(u) \wedge \mu_1(v)$$

$$\rho_2(u, v) \leq \mu_2(u) \wedge \mu_2(v)$$

for all $(u, v) \in E$ and denoted by $B = \{(u, v), [\rho_1(u, v), \rho_2(u, v)] : (u, v) \in E\}$, where the function $\rho_1 : E \to [0, 1]$ and $\rho_2 : E \to [0, 1]$, such that $\rho_1(u, v) \leq \rho_2(u, v)$ for $(u, v) \in E$.

Definition 2.3. *[1, 4]* An interval-valued fuzzy graph of the graph $G^* = (V, E)$ is a pair $G = (A, B)$ where

(1) $A = [\mu_1, \mu_2]$ is an interval-valued fuzzy set on V.

(2) $B = [\rho_1, \rho_2]$ is an interval-valued fuzzy relation on V, such that

$$\rho_1(u, v) \leq min\{\mu_1(u), \mu_1(v)\}$$

and for all

$$(u, v) \in E.$$
$$\rho_2(u, v) \leq min\{\mu_2(u), \mu_2(v)\}$$

2

Definition 2.4. *[1]* An edge $e = (x, y)$ of an interval-valued fuzzy graph is called effective edge if $\rho_1(x, y) = min\{\mu_1(x), \mu_1(y)\}$ and $\rho_2(x, y) = min\{\mu_2(x), \mu_2(y)\}$. The degree of a vertex can be generalized in different ways for an interval-valued fuzzy graph $G = (A, B)$ of $G^* = (V, E)$. The effective degree of a vertex v in an interval-valued fuzzy graph $G = (A, B)$ is defined to be sum of the weights of the effective edges incident at v and it is denoted by $d_E(v)$. The minimum effective edges degree of G is $\delta_E(G) = min\{d_E(v)|v \in V\}$. The maximum effective degree of G is $\Delta_E(G) = max\{d_E(v)|v \in V\}$.

Definition 2.5. *[1]* Two vertices u and v are said to be neighbors in interval-valued fuzzy graph $G = (A, B)$ of $G^* = (V, E)$ if $\rho_1(u, v) = \mu_1(u) \wedge \mu_1(v)$ and $\rho_2(u, v) = \mu_2(u) \wedge \mu_2(v)$. A vertex subset $N(v) = \{u \in V : v \text{ adjacent to } u\}$ is called the open neighborhood set of a vertex v and $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood set of v. The neighborhood degree of a vertex v in an interval-valued fuzzy graph $G = (A, B)$ is defined to be sum of the weights of the vertices adjacent to v and it is denoted by $d_N(v)$, that is mean that $d_N(v) = |N(v)|$. The minimum neighborhood degree of G is $\delta_N(G) = min\{d_N(v)|v \in V\}$. The maximum neighborhood degree of G is $\Delta_N(G) = max\{d_N(v)|v \in V\}$. The neighborhood degree of a vertex v in an interval-valued fuzzy graph $G = (A, B)$ is defined to be the sum of the weights of the vertices which adjacent to v and is denoted by $d_s(v)$.

Definition 2.6. *[6]*An interval-valued fuzzy graph $G = (A, B)$ is said to be complete interval- valued fuzzy graph if $\rho_1(v_i, v_j) = min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\rho_2(v_i, v_j) = min\{\mu_2(v_i), \mu_2(v_j)\}$ for all $v_i, v_j \in V$ and is denoted by K_μ. The complement of an interval-valued fuzzy graph $G = (A, B)$ is an interval-valued fuzzy graph $\overline{G} = (\overline{V}, \overline{E})$ where

(i) $\overline{V} = V$;
(ii) $\overline{\mu_1} = \mu_1$; $\overline{\mu_2} = \mu_2$ for all vertices;
(iii) $\overline{\rho_1}(u, v) = min\{\mu_1(u), \mu_1(v)\} - \rho_1(u, v)$ and $\overline{\rho_2}(u, v) = min\{\mu_2(u), \mu_2(v)\} - \rho_2(u, v)$ for all $u, v \in E$.

Definition 2.7. *[6]* An interval-valued fuzzy graph $G = (A, B)$ is said to bipartite if the ver- tex set V of G can be partitioned into two non empty sub sets V_1 and V_2 such that $V_1 \cap V_2 = \varphi$. A bipartite interval-valued fuzzy graph $G = (A, B)$ is said to be complete bipartite interval- valued fuzzy graph if $\rho_1(v_i, v_j) = min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\rho_2(v_i, v_j) = min\{\mu_2(v_i), \mu_2(v_j)\}$ for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{m,n}$, where $|V_1| = m, |V_2| = n$.

Definition 2.8. *[3]* A vertex $u \in V$ of an interval-valued fuzzy graph $G = (A, B)$ is said to be an isolated vertex if $\rho_1(v, u) = 0$ and $\rho_2(v, u) = 0$ for all $v \in V$. That is, $N(u) = \varphi$.

Definition 2.9. *[10]* An interval-valued fuzzy graph $G = (A, B)$ is said to be strong interval- valued fuzzy graph, if $\rho_1(u, v) = min(\mu_1(u), \mu_1(v))$ and $\rho_2(u, v) = min(\mu_2(u), \mu_2(v))$ for all $(u, v) \in E$.

Definition 2.10. *[1]* An interval-valued fuzzy graph $H = (A_1, B_1)$ is said to be an interval-valued fuzzy sub graph of $G = (A, B)$, if $A_1 \subseteq A$ and $B_1 \subseteq B$. That is $\mu'_1 \le \mu_1$, $\mu'_2 \le \mu_2$ and $\rho'_1 \le \rho_1$; $\rho'_2 \le \rho_2$.

Definition 2.11. *[3]* Let $G = (A, B)$ be an interval valued fuzzy graph and let $S \in V(G)$. Then a vertex subset S of G is said to be independent set if $\rho_1(u, v) < \mu_1(u) \wedge \mu_1(v)$ and $\rho_2(u, v) < \mu_2(u) \wedge \mu_2(v)$ or $\rho_1(u, v) = 0$ and $\rho_2(u, v) = 0$ for all $u, v \in S$. An independent vertex subset S of an interval valued fuzzy graph $G = (A, B)$ is said to be maximal indepen- dent set if $S \cup \{u\}$ is not independent $\forall u \in V - S$. The maximum fuzzy cardinality among all maximal independent sets in interval valued fuzzy graph G is called the independence number of G and is denoted by $\beta_0(G)$.

Definition 2.12. *[3]* Let $G = (A, B)$ be an interval-valued fuzzy graph and let $u, v \in V(G)$. Then we say that u dominates v or v dominates u if (u, v) is a effective edge, i.e $\rho_1(u, v) = min \mu_1(u), \mu_1(v)$ and $\rho_2(u, v) = min \mu_2(u), \mu_2(v)$. A vertex sub set $(D \subseteq V)$ of $V(G)$ is called dominating set in an interval-valued fuzzy graph G, if for every $v \in V - D$ there exists $u \in D$ such that (u, v) is effective edge. A dominating set D of an interval-valued fuzzy graph G is called minimal dominating set if $D - \{u\}$ is not dominating set for every $u \in D$. A minimal dominating set D with $|D| = \gamma(G)$ is denoted by $\gamma - set$.

3 Connected Domination Of An Interval-Valued Fuzzy Graph.

Definition 3.1. *Let $G = (A, B)$ be interval-valued fuzzy sub graph of graph $G^* = (V, E)$ and let $v; u \in V$. Then G is a connected interval-valued fuzzy graph G if for every $v; u \in V(G)$. Thence is a path $u - v$ spanning u and v.*

Definition 3.2. *A dominating set D in an interval-valued fuzzy graph G is said to be con- nected dominating set if the induced interval-valued fuzzy sub graph $< D >$ is connected and is denoted by D_c.*

Definition 3.3. *A connected dominating set D_c of an interval-valued fuzzy graph G is said to be minimal connected dominating set if for every $u \in D_c$, $D_c - \{u\}$ is not connected dominating set of G.*

Definition 3.4. *The minimum fuzzy cardinality of all minimal connected dominating set of interval-valued fuzzy graph G is called the connected domination number of G and denoted by $\gamma_c(G)$.*

Definition 3.5. *The maximum fuzzy cardinality of all minimal connected dominating set of interval-valued fuzzy graph G is called the upper connected domination number of G and denoted by $\Gamma_c(G)$.*

Remark 3.6. *A minimal connected dominating set D_c of an interval-valued fuzzy graph G, with $|D_c| = \gamma_c(G)$ is the minimal connected dominating set of G and is denoted by $\gamma_c -$set.*

Example 3.7. *For the interval-valued fuzzy graph G given in Figure 1, such that all edges of G are effective.*
We see that, $D_{c1} = \{v_2, v_4\}$ and $D_{c2} = \{v_3, v_4\}$ are minimal connected dominating sets.

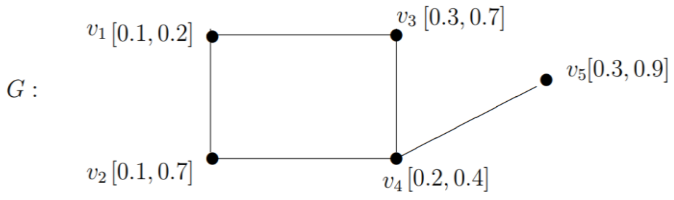


Fig 1

Since $D_{c1} = \{v_2, v_4\}$ is $\gamma_c -$ set of G, then $\gamma_c(G) = 1.3$. And $D_{c2} = \{v_3, v_4\}$ is the maximum connected dominating set of G, then $\Gamma_c(G) = 1.4$.

Theorem 3.8. *Every connected dominating set of an interval-valued fuzzy graph G is a dom-inating set of G. But the converse is need not true.*

Proof. Let $G = (A, B)$ be an interval-valued fuzzy graph and let D_c be a connected dominat- ing set of G by definition of the connected dominating set of an interval-valued fuzzy graph G, D_c is dominating set of G.
The converse of the above theorem need not be true.
For the interval-valued fuzzy graph given in Figure 1, we have $D = \{v_1, v_5\}$ is dominating set of G, but it is not connected. □

Corollary 3.9. *For any interval-valued fuzzy graph G. $\gamma(G) \leq \gamma_c(G)$.*
Proof. Since every connected dominating set of an interval-valued fuzzy graph G is domi- nating set of G, by the above Theorem. Hence $\gamma(G) \leq \gamma_c(G)$. □

Theorem 3.10. *Let $G = (A,B) = K_\mu$ be any complete interval-valued fuzzy graph and let D_c be a connected dominating set of G. Then $V - D_c$ is a connected dominating set of k_μ.*
Proof. Let $G = K_\mu$ be a complete interval-valued fuzzy graph and let D_c be a connected dominating set of G. Then all edges in G are strong edges and each vertex in G dominates to all the other vertices of V. Hence a connected dominating set D_c contain one vertex say v. Since all edges in G are strong arcs and each vertex in D_c dominates to all the other vertices in G. Therefore $V - D_c = V - \{v\}$ is also a connected dominating set of G.

Theorem 3.11. *Let G be any complete interval-valued fuzzy graph and let G be the comple- mentary ~~of~~ G. Then connected dominating set in G is not exists.*
Proof. Let G be any complete interval-valued fuzzy graph and let G ~~be~~ a complement of G. Then all edges of G are a strong edges and every vertex in G dominates to all the other vertices of G. By the above Theorem, connected dominating is exists. Since G be the complementary of G. Then G is null interval-valued fuzzy graph. Therefore the connected dominating set of G is not exists.
Corollary 3.12. *For any complete interval-valued fuzzy graph $G = K_\mu$. Then*

$$\gamma(G) = \gamma_c(G).$$

Theorem 3.13. *A connected dominating set D_c of interval-valued fuzzy graph G is minimal connected dominating set ~~if~~ and only if one of the folloing condition holds:*
(i) $N(v) \cap D_c = \varphi$
(ii) *There is a vertex $u \in V - D_c$, such that $N(u) \cap D_c = \{v\}$.*

4

Proof. Let G be interval-valued fuzzy graph with D_c is $\gamma -$ *set* of G and let $v \in D_c$. Then $D_c - \{v\}$ is not a connected dominating set of G and there exists a vertex $u \in V - \{D_c - \{v\}\}$, such that u is not dominated by any vertex of $D_c - \{v\}$. If $u = v$, then $N(v) \cap D_c = \varphi$. If $u \neq v$, then $N(u) \cap D_c = \{v\}$.

Conversely: suppose that D_c is a connected dominating set and for each a vertex $v \in D_c$ one of two conditions holds. Suppose that D_c is not minimal connected dominating set of G, then there exsits a vertex $v \in D_c$, such that $D_c - \{v\}$ is a connected dominating set of G. Hance v is adjacemt to at least one vertex in $D_c - \{v\}$, then the condition one does not hold, which a contradiction. If $D_c - \{v\}$ is connected dominating set, then every vertex in $V - D_c$ is adjacent to at least one vertex in $D_c - \{v\}$, then the condition two dose not hold, which a contradiction. So D_c is minimal connected domanating set of interval-valued fuzzy graph G.

Theorem 3.14. *Let $G = (A, B)$ be any connected interval-valued fuzzy graph of $G^* = (V, E)$ and $G_1 = (A_1, B_1)$ be any maximum spanning tree of G. Then every connected dominating set of G_1 is also a connected dominating set of G and $\gamma_c(G) \leq \gamma_c(G_1)$.*

Proof. Let D_c be a connected dominating set of a maximum spanning tree G_1 of an interval- valued fuzzy graph G. Since G_1 is maximum spanning tree of an interval-valued fuzzy graph G with a vertex set $V(G_1) = V(G)$. Then for every vertex $v \in V - D_c$ there exists a vertex $u \in D_c$ such that u dominates v. Hence D_c is a connected dominating set of G. Hence $\gamma_c(G) \leq \gamma_c(G_1)$.

Example 3.15. *For the interval-valued fuzzy graph G and its maximum spanning interval- valued fuzzy sub graph G_1 in the following two Figures, such that each edges of G and G_1 are strong edges.*

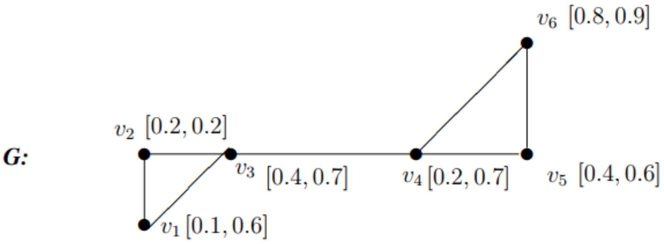


Fig 2

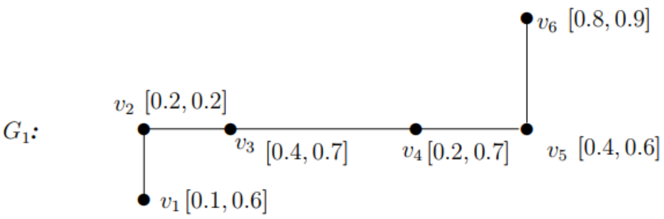


Fig 3

We see that, $D_c(G) = \{v_3, v_4\}$ is a connected dominating set of an interval-valued fuzzy graph G. Then a connected domination number of G, $\gamma_c(G) = 1.4$.
Also we see that, $D_c(G_1) = \{v_2, v_3, v_4, v_5\}$ is a connected dominatig set of an interval-valued fuzzy sub graph G_1. Then a connected domination number of G_1, $\gamma_c(G_1) = 2.5$.
Hence $\gamma_c(G) \leq \gamma_c(G_1)$.

Remark 3.16. *Since a connected dominating set $D_c = n - 2$ for any strong path P_n, strong cycle C_n and bipartite $K_{n1,n2}$ interval-valued fuzzy graph G with $n \geq 3$, where n the number of vertices of G and $n_1 + n_2 = n$.*

Theorem 3.17. *For any strong path P_n interval-valued fuzzy graph G with $n \geq 3$. Then*

$$|D_c| = \gamma_c(G) = \sum_{i=1}^{n-2} |v_i|$$

for $v_i \in V$.

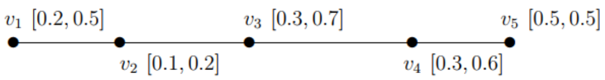
5

Proof. Let $G = P_n$ be a strong path interval-valued fuzzy graph with $n \geq 3$, where n the number of vertices of G. Since a path P of length n is a sequence of vertices $v_0, v_1, \ldots, v_n$, where $\mu(v_{i-1}, v_i) > 0$, $i = 1, 2, ..., n$. Then connected dominating set $D_c = n-2$ by the above Remark, i.e $D_c = \{v_1, v_2, ..., v_{n-1}\}$ except v_o, v_n. Hence $|D_c| = |v_1| + \ldots + |v_{n-1}| = \sum_{i=1}^{n-2} |v_i|$

Therefore

$$|D_c| = \gamma_c(G) = \sum_{i=1}^{n-2} |v_i|$$

Example 3.18. *For interval-valued fuzzy graph given in Figure (4), such that all edges of G are strong.* □



G: ***Fig 4***

From the above Figure, we have $D_c = \{v_2, v_3, v_4\}$ is connected dominating set of G. Then connected domination number

$$|D_c| = \sum_{i=1}^{n-2} |v_i| = \sum_{i=1}^{3} |v_i|$$

$$= |v_2| + |v_3| + |v_4| = 1.9$$

Theorem 3.19. *For any bipartite $K_{n1,n2}$ interval-valued fuzzy graph G with $n \geq 3$. Then*

$$|D_c| = \gamma_c(G) = \sum_{i=1}^{n-2} |v_i|,$$

for $v_i \in V$.

Proof. Let $G = K_{n1,n2}$ be bipartite interval-valued fuzzy graph with $n \geq 3$, where $n = n_1 + n_2$ such that n_1 and n_2 the number of vertices V_1 and V_2 of G respectively. Since a vertex set V_1 dominates a vertex set V_2 and conversly any bipartite which it is independent dominating set of G, but it is not connected dominating set of G. Hence connected dominating set D_c in the bipartite is the same connected dominating set in the path. By the above Theorem, then

$$|D_c| = \gamma_c(G) = \sum_{i=1}^{n-2} |v_i|$$

Example 3.20. *Consider the interval-valued fuzzy graph G, shown in Figure (5) such that all edges of G are effective.*

G:

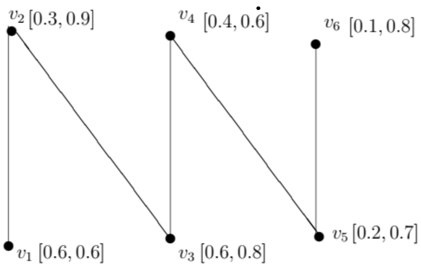


Fig 5

From the Figure, we see that $D_c = \{v_2, v_3, v_4, v_5\}$ *is a connected dominating set of G. Since*

$$|D_c| = \sum_{i=1}^{n-2} |v_i|,$$

for $v_i \in V$. *Then*

$$|D_c| = \gamma_c(G) = \sum_{i=1}^{4} |v_i|$$

$$= |v_2| + |v_3| + |v_4| + |v_5| = |0.8 + 0.6 + 0.6 + 0.75| = 2.75$$

.

Remark 3.21. *For any bipartite, a astrong cycle or a strong path interval-valued fuzzy graph G with* $n \geq 4$, *where n the number of vertices of G. Then*

(1) $\gamma_c(G) \geq p - \Delta_N(G)$, *if* $\mu_1(u) = \mu_2(u)$ *to each* $u \in V - D_c$ *and* $\mu_1(v) \leq \mu_2(v)$ *to each* $v \in D_c$.

(2) $\gamma_c(G) \leq p - \Delta_N(G)$, *if* $\mu_1(u) \leq \mu_2(u)$ *to each* $u \in V - D_c$ *and* $\mu_1(v) = \mu_2(v)$ *to each* $v \in D_c$.

(3) $\gamma_c(G) + \frac{1}{2} \geq p - \Delta_N(G)$ *for any interval-valued fuzzy graph G, namely* $\mu_1(u) \leq \mu_2(u)$ *to each* $u \in V$.

Theorem 3.22. *For any bipartite interval-valued fuzzy graph* $G = K_{n_1,n_2}$ *with* $n \geq 4$, $n = n_1 + n_2$, *where* n_1 *and* n_2 *the number of vertices* V_1 *and* V_2 *respectively. Then*

$$\frac{p}{\Delta_N(G) + 1} \leq \gamma_c(G)$$

Proof. Let G be bipartite interval-valued fuzzy graph with $n \geq 4$, $n = n_1 + n_2$, where n_1 and n_2 the number of vertices V_1 and V_2 respectively with $\gamma_c - set$ and let v be a vertex of G with $\Delta_N(v) = d_N(v)$. Since $\gamma_c(G) \leq p - \Delta_N(G)$ or $\gamma_c(G) \geq p - \Delta_N(G)$. Then $p - \gamma_c(G) \leq \Delta_N(G)$ or $p - \gamma_c(G) \geq \Delta_N(G)$. Thus clear that

$$p - \gamma_c(G) \leq \gamma_c(G).\Delta_N(G).$$ □

$$p \leq \gamma_c(G).\Delta_N(G) + \gamma_c(G)$$

$$p \leq \gamma_c(G).(\Delta_N(G) + 1)$$

$$\frac{p}{\Delta_N(G) + 1} \leq \gamma_c(G)$$

Example 3.23. *From the above figure (5), we have* $p = 8.2$, $\gamma_c(G) = 6.85$ *and* $\Delta_N(G) = 1.45$. *Then*

7

$$\frac{p}{\Delta_N(G)+1} \leq \gamma_c(G)$$

$$\frac{8.2}{1.45+1} \leq 6.85$$

$$3.3469 < 6.85$$

Theorem 3.24. *For any complete bipartite interval-valued fuzzy graph, $G = k_{m,n}$. Then*

$$\gamma_c(G) = min\{|x| : x \in V_1\} + min\{|y| : y \in V_2\}.$$

Proof. Let $G = k_{m,n}$ be complete bipartite interval-valued fuzzy graph, where m and n are the number of vertices V_1 and V_2 respectively. Then all edges in G are effective and each vertex in V_1 dominates to all vertices in V_2 and each vertex in V_2 dominates to all vertices in V_1. Hence a connected dominating set of G contain exactly two vertices x,y where $x \in V_1$ and $y \in V_2$. Hence x has the minimum membership value in V_1 and y has the minimum membership value of V_2 Therefour,

$$\gamma_c(G) = min\{|x| : x \in V_1\} + min\{|y| : y \in V_2\}.$$

□

Example 3.25. *Consider the interval-valued fuzzy graph G, shown in Figure (6), such that all edges of G are effective.*

G:

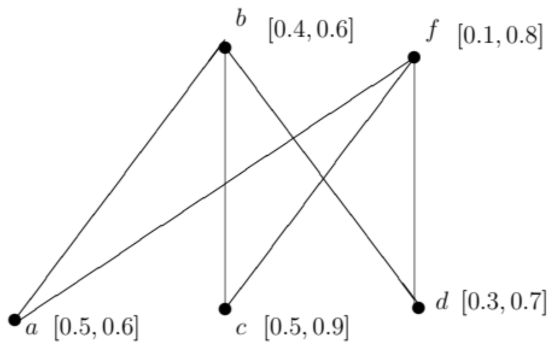


Fig 6

From the Figure, we see that a vertex subset $\{a, b\}$, $\{a, f\}$, $\{c, b\}$, $\{c, f\}$, $\{d, b\}$ and $\{d, f\}$ are minimal connected dominating set of an interval-valued fuzzy graph G. Then minimum connected dominating set of $G = \{a, b\}$. Hence $\gamma_c(G) = 1.15$

Remark 3.26. *For any complete bipartite interval-valued fuzzy graph $G = k_{m,n}$ with γ_c−set. Then $V - D_c$ is a connected dominating set of $k_{m,n}$.*

Example 3.27. *From the above Example in figure (6), since $D_c = \{a,b\}$ is γ_c−set. Then $V - D_c = \{c,f,d\}$ is a connected dominating set of G.*

8

4 Conclusion

In this paper, the concepts of connected dominating set and connected domination number are defined on interval-valued fuzzy graphs and also applied for the various types of interval- valued fuzzy graphs and suitable examples have given. Uper and lower bounds of $\gamma_c(G)$ in interval-valued fuzzy graphs G are obtained. We have done some results with examples and Relations of connected domination number with known parameters in interval-valued fuzzy graph were discussed with the suitable examples.

REFERENCES

[1] M. Akram and W. A. Dudek, ”Interval-valued fuzzy graphs”, Cmput. Math. Appl. 61, (2011), 289-299.
[2] E. J. Cockayne and S. T. Hedetniemi, Towards a theory of domination in graphs, Net- works 7 (1977) 247-261.
[3] P. Debnath, Domination in interval-valued fuzzy graphs, Annals of Fuzzy Mathematics and Informatics. 6(2), (2013), 363-370.
[4] H. Ju and L. Wang, Interval-valued fuzzy sub semi groups and subgroups associated by interval-valued fuzzy graphs, in Proceedings of the WRI Global Congress on Intelligent Systems (GCIS 09), pp. 484487, Xiamen, China, May 2009.
[5] O. T. Manjusha, M. S. Sunitha, connected domination in fuzzy graphs using strong arcs, Annals of Fuzzy Mathematics and Informatics, (2015), 1-16.
[6] H. Rashmanlou and Y. B. Jun, Complete interval valued Fuzzy graphs, Annals of Fuzzy Mathematics and Informatics, 3(1), (2013),107-130
[7] A. Rosenfeld, Fuzzy graphs, In. L. A. Zadeh, K. S. Fu and M. Shimura, (Eds). ”Fuzzy Sets and their Applications”, Academic Press, New York, (1975), 77-95.
[8] E. Sampathkumar, H. B. Walikar, The connected domination number of a Graph, Jour.Math.Phy.Sci. 13(6), (1979).
[9] N. Sarala, P. Janaki, Connected domination number of fuzzy digraphs, research article IJSRR, 8(2), (2019), 4096-4100.
[10] Sarala N, Kavitha T. Strong (Weak) Domination in Interval Valued Fuzzy Graph. Dis- covery, 2016, 52(248), 1626-1634.
[11] A. Somasundaram, S. Somasundaran, ”Domination in fuzzy graphs-I”, Pattern Recog- nition Letters, 19, (1998), 787-791.
[12] A. Somasundaram, ”Domination in fuzzy graphs-II”, Journal of Fuzzy Mathematics, 20, (2000), 281-289.
[13] L. A. Zadeh, The concept of a linguistic and application to approximate reasoning I, Inform. Sci. 8, (1975), 149-249.